IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

EEE PART I: MEng, BEng and ACGI

DEVICES AND FIELDS

Corrected Copy

Wednesday, 26 May 10:00 am

Time allowed: 2:00 hours

There are FIVE questions on this paper.

There are two sections. Answer THREE questions including at least ONE question from each section.

QUESTION 1 is COMPULSORY

Use a separate answer book for each section.

All questions carry equal marks

10am: SECTION A: Q1, Q2, Q3
SECTION B: Q4, Q5.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K. Fobelets, E.M. Yeatman
Second Marker(s) : K.D. Leaver, D. Popovic

Special information for invigilators: Q1 is obligatory

Information for candidates

permittivity of free space : $\varepsilon_o = 8.85 \times 10^{-12}$ F/m

permeability of free space : $\mu_o = 4\pi \times 10^{-7}$ H/m

1. **This question is obligatory!**

a) Sketch the energy band diagram of an ohmic contact on n-type Si when the potential on the metal is positive with respect to the potential on the semiconductor. Include conduction, valence band, Fermi level and applied voltage in the graph. [2]

b) A metal is to be chosen to make a Schottky contact on p-type Si. What condition must the metal work function satisfy? [1]

c) Give the names of two 2-terminal devices that can be used as rectifiers. [2]

d) Sketch the minority carrier concentrations in both p and n regions of a forward biased pn junction when the doping concentration in the p region is much higher than the doping concentration in the n-region ($N_A >> N_D$) and the n and p regions of the diode are longer than the diffusion length of the carriers. Ensure that the relative magnitudes of the minority carrier concentration at both sides of the junction are correct. [4]

e) Draw the I_{DS}-V_{DS} curve of an n-channel MOSFET for one gate voltage ($V_{GS}>V_{th}$) and sketch the cross section of the MOSFET including depletion and inversion layers for the following operation conditions:
i) triode region
ii) pinch off.
Indicate the regions i) & ii) on the I_{DS}-V_{DS} curves. [4]

f) Draw a cross section of an npn bipolar transistor in the active mode and indicate the principal electron and hole currents flowing through the device. [3]

g) Find the work done to move a charge of +2 C in the x-y plane from (x,y) = (4,4) to (1,1) if there is a uniform electric field in the $+x$ direction of 5 V/m. [2]

h) Two thin parallel wires in air are each carrying constant currents of 1 A in the same direction. The wires are 2 m apart. Find the magnetic flux density at a point mid-way between the two wires. [2]

2.

a) What are the electrical differences between an n-channel enhancement mode and a p-channel depletion mode MOSFET? [4]

b) Sketch the cross-section of an n-channel enhancement mode MOSFET and an npn planar BJT (all contacts on the surface). [6]

c) For both devices in b) explain how varying the applied voltage between two of the terminals of the devices controls the current flowing into the third terminal. [4]

d) The capacitance-voltage characteristic of a MOSFET is given in figure 2.1. Give the type of the MOSFET (n or p-channel) and determine the thickness of the oxide and the maximum width of the depletion layer in the substrate (bulk).
$\varepsilon_0 = 8.85\ 10^{-14}$ F/cm, $\varepsilon_{Si} = 11.8$ and $\varepsilon_{SiO2} = 3.9$. [6]

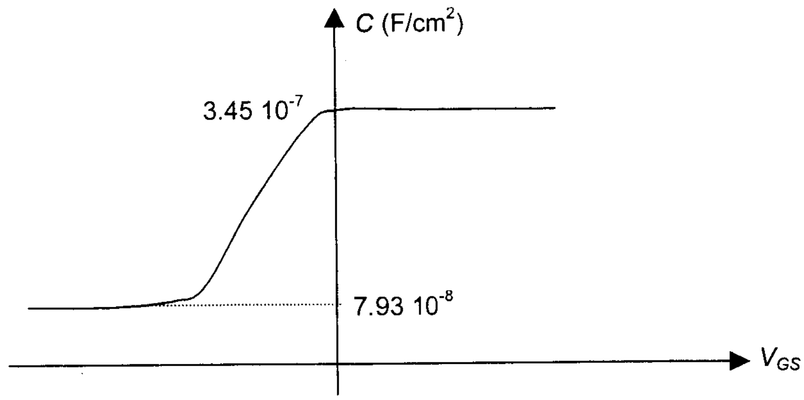


Figure 2.1: The measured capacitance-voltage characteristics of the gate-oxide-semiconductor contact of the MOSFET.

3.

a) i) Which geometrical parameter must be small for fast operation of a MOSFET?
ii) Which geometrical parameter must be small for high gain in a BJT? [2]

b) Give the type (drift or diffusion) of the currents, I_{DS} in a MOSFET and I_C in a BJT. Explain your answers briefly. [4]

c) Draw the small-signal equivalent circuit of a
i) MOSFET in saturation
ii) BJT in active mode in common emitter configuration
Give the definition of all the parameters in the equivalent circuits. [6]

d) Calculate the excess minority carrier charge per unit area in the base of an npn BJT at 300 K with $\gamma = 1$, no recombination and negligible depletion into the base region.

The base width is 1 μm,
The base doping is 10^{16} cm^{-3}
The base-emitter voltage is $V_{BE} = 0.6$ V
The base-collector voltage is $V_{BC} = -1$ V
$\frac{kT}{q} = 0.025\,V$
$q = 1.6\ 10^{-19}\ C$
$n_i = 1.45\ 10^{10}\ cm^{-3}$.
Remember that the carrier concentration, c, injected across a junction is given by:
$$c = c_0 e^{\frac{|q|V}{kT}}$$
with c_0 the equilibrium carrier concentration and V the voltage across the junction.
Make the necessary assumption to simplify the calculation. [8]

4.

a) Two parallel conducting square plates of area A are separated by an air gap of length t, and carry a charge of $+Q$ and $-Q$ respectively. Using Gauss' law, derive the electric flux density D, and hence the electric field strength E, between the plates. Assume that $\sqrt{A} >> t$. [4]

b) From your answer to (a), calculate the capacitance C of the pair of plates. [4]

c) A dielectric material is now added between the two plates which has a linearly graded relative permittivity, as shown in Fig 4.1. Calculate the electric field distribution $E(x)$ between the plates. [4]

d) From your answer to (c), calculate the voltage between the plates, and hence show that the capacitance can be given by $C = \dfrac{(\varepsilon_{r2} - \varepsilon_{r1})\varepsilon_o A}{t \ln(\varepsilon_{r2}/\varepsilon_{r1})}$. [4]

e) Show that the solution to (d) reduces to the expected value if $\varepsilon_{r2} = \varepsilon_{r1}$. [4]

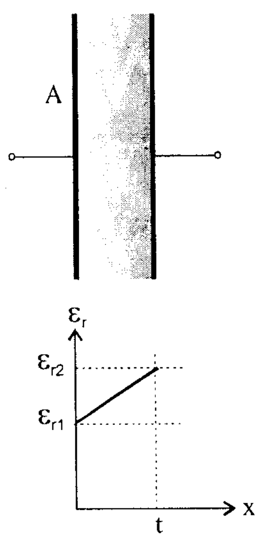


Figure 4.1

5.

a) Two wires in air, lying along the x and y axes as shown in Fig. 5.1(a), carry equal currents I in the $+x$ and $+y$ directions respectively. What will be the direction of the magnetic flux density B in the ($x>0$, $y<0$) quadrant of the x-y plane? [4]

Where, if anywhere, in the x-y plane, will $B = 0$? [4]

b) The two wires are now rotated -45° about the z axis as shown in Fig. 5.1(b). Derive an expression for the magnitude of the magnetic flux density $|B(x)|$ along the x axis. [4]

What will be the direction of B in this case along the z axis? [4]

Derive an expression for the magnitude of the magnetic flux density $|B(z)|$ along the z axis. [4]

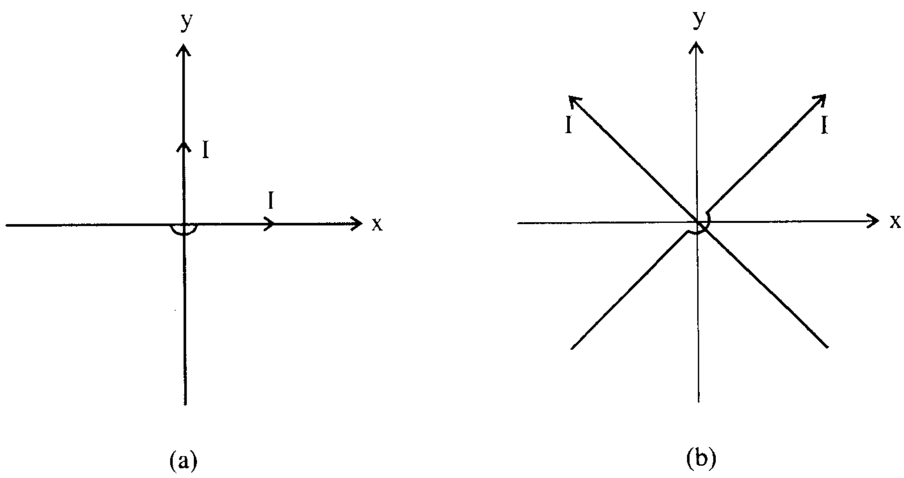
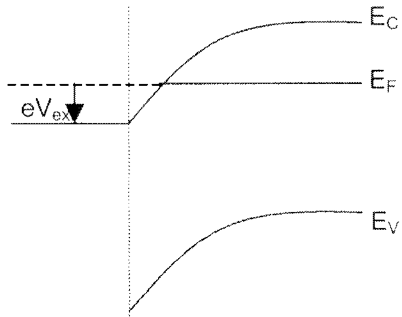


Figure 5.1

1. **This question is obligatory!**

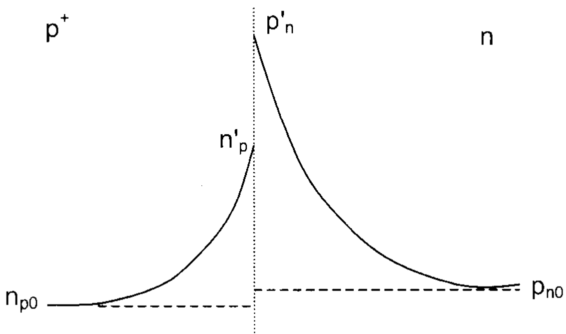
a) [2]



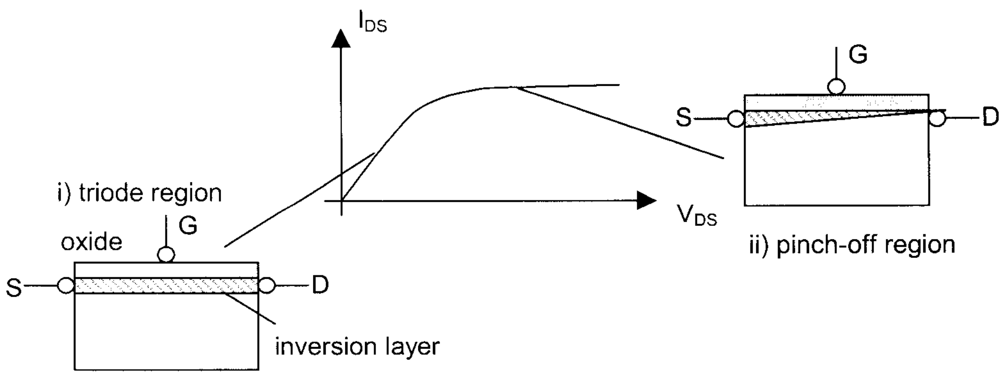
b) $\phi_m < \phi_s$ with ϕ_m the metal workfunction and ϕ_s the semiconductor workfunction. [1]

c) pn junction (diode), schottky junction (diode). [2]

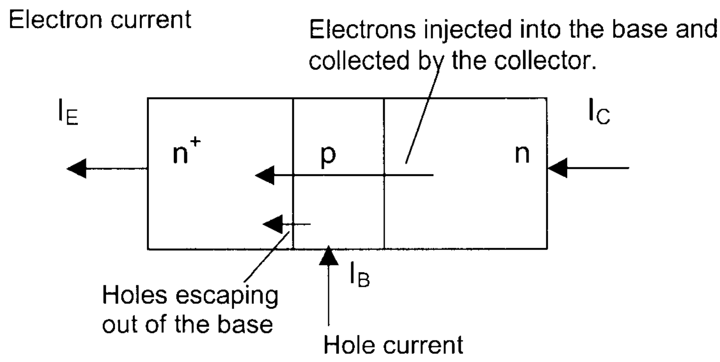
d) [4]



e) [4]



f) [3]



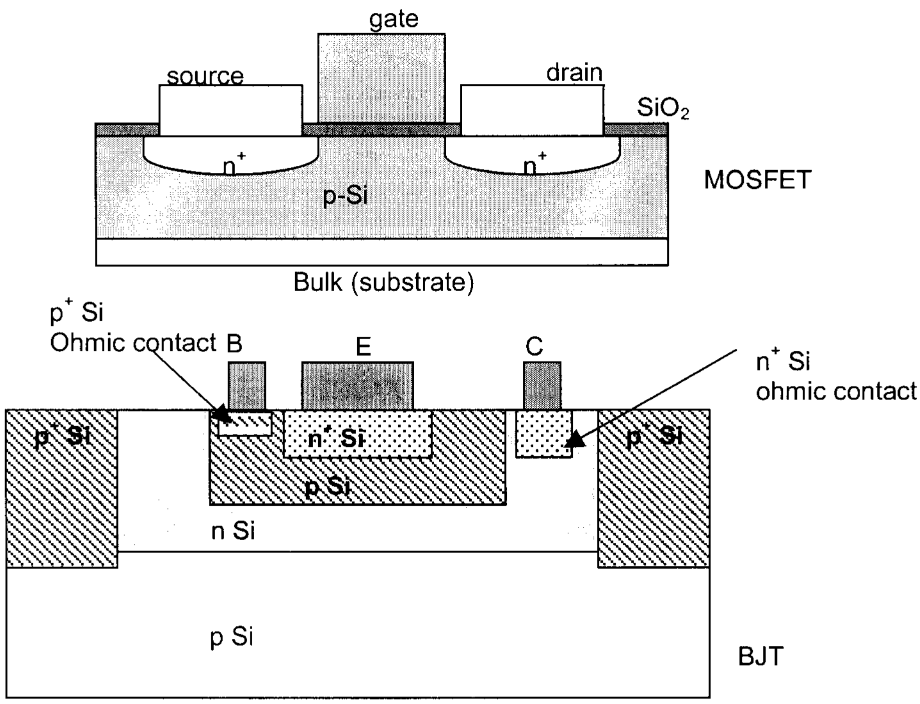
g) $W = Q \cdot \Delta V = Q \cdot E \cdot \Delta x = 2(5)(3) = 30$ J. [2]

h) Since the currents are parallel, B rotates about each with the same sense, therefore along the centre line the contributions from the two wires are equal and opposite and so $\underline{B = 0}$. [2]

2.

a) n-channel enhancement mode : carriers in channel are electrons and no channel exists at zero gate bias.
p-channel depletion mode : carriers in channel are holes and a channel exists at zero gate bias. [4]

b)



[6]

c) MOSFET: the voltage on the gate controls the density of charges in the channel. The current between source and drain is directly proportional to the density of charges in the channel.
BJT: the base current supplied by a series connection of a resistance and base voltage supply controls charge neutrality in the base. If the base current decreases, the internal potentials across the pn junctions have to adapt to retain a neutral base. [4]

d) n-channel MOSFET
The maximum capacitance = accumulation capacitance = C_{ox}

$$C_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{t_{ox}}$$

$$t_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{C_{ox}} = \frac{8.85\,10^{-14}\;3.9}{3.45\,10^{-7}} = 10^{-6} cm = 10 nm$$

The minimum capacitance is due to the series connection of the oxide capacitance and the depletion capacitance associated to the maximum depletion width t_{depl}.

[E1.3]

$$C_d = \frac{\varepsilon_0 \varepsilon_{SiO_2}}{t_{depl}}$$

$$C_{\min} = \frac{C_d C_{ox}}{C_d + C_{ox}} = 7.93\,10^{-8}\,F/cm^2$$

$$C_d = \frac{C_{ox} C_{\min}}{C_{ox} - C_{\min}} = \frac{7.93\,10^{-8}\;3.45\,10^{-7}}{3.45\,10^{-7} - 7.93\,10^{-8}} = 1.03\,10^{-7}\,F/cm^2$$

$$t_{depl} = \frac{\varepsilon_0 \varepsilon_{SiO_2}}{C_d} = \frac{8.85\,10^{-14}\;11.8}{1.03\,10^{-7}} \approx 10^{-5}\,cm = 101.4\,nm$$

[6]

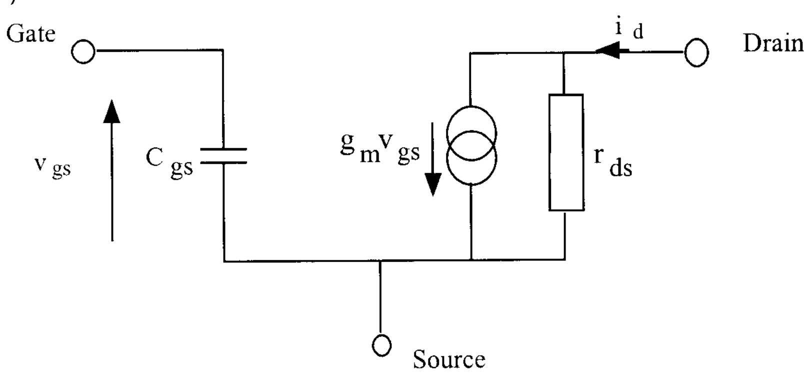
3.

a) i) MOSFET: gate length, ii) BJT base width [2]

b) MOSFET: drift current. Once the gate has created a channel, the current between source and drain through the channel see one carrier type environment and the electric field applied across the channel will result in movement of the *majority* carriers.
BJT: Diffusion currents. Under forward bias majority carriers are injected across the junction. This injection of carriers creates an increased minority carrier concentration at the junction in the opposite material side. The *minority* carrier gradients in the p and n region will then cause diffusion currents. [4]

c)

i) MOSFET in saturation



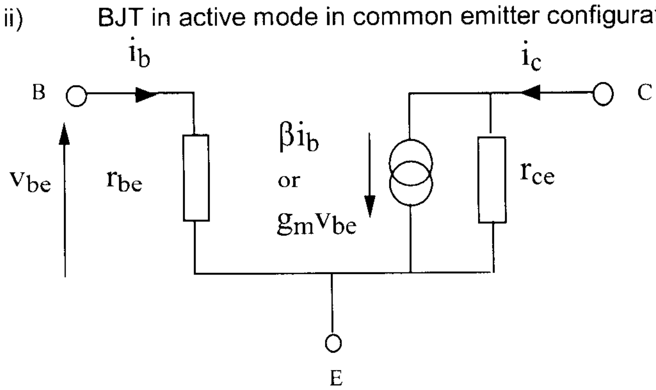
g_m vgs: voltage controlled current source with g_m transconductance

$g_m = \frac{dI_{DS}}{dV_{GS}}$.

$r_{ds} = \left(\frac{dI_{DS}}{dV_{DS}}\right)^{-1}$ output resistance

$C_{gs} = \frac{dQ_G}{dV_{GS}}$ input capacitance

ii) BJT in active mode in common emitter configuration



g_m v_{be} or βi_b, Voltage or current controlled current source

Current gain $\beta = i_c/i_b$, or transconductance $g_m = \frac{dI_C}{dV_{BE}}$

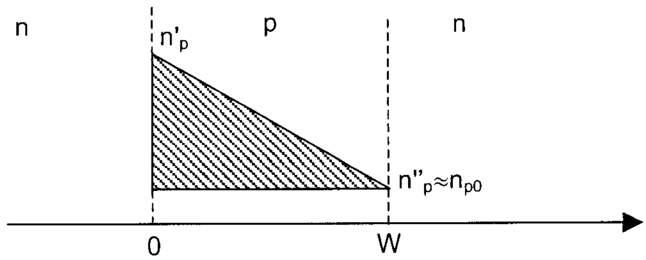
Input resistance, r_{be}

$r_{be} = \left|\frac{dV_{BE}}{dI_B}\right| = \left|\frac{dV_{BE}}{dI_C}\frac{dI_C}{dI_B}\right| = \beta/g_m$

Output resistance, r_{ce}

$$r_{ce} = \left|\frac{dV_{CE}}{dI_C}\right|$$ [6]

d) The excess minority carrier concentration in the base under the given conditions is given schematically in the figure:



Notice: linear variation in minority carrier concentration in the base. The reverse bias minority carrier concentration at the B-C junction is assumed equal to the equilibrium concentration.

The charge in the base is proportional to the crossed area in the figure.

$$Q_n \approx \frac{qW(n'_p - n_{p0})}{2} = \frac{qWn_{p0}\left(e^{\frac{|q|V_{BE}}{kT}} - 1\right)}{2} = \frac{qWn_i^2\left(e^{\frac{|q|V_{BE}}{kT}} - 1\right)}{2N_A} = 0.8\,10^{-19}\,10^{-4}\left(e^{\frac{0.6}{0.025}} - 1\right)\frac{(1.45\,10^{10})^2}{10^{16}} = 4.46\,10^{-9}\,Ccm^{-2}$$

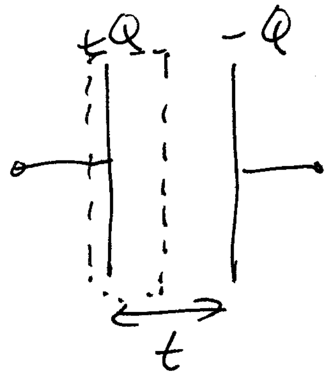
without the assumption

$$Q_n \approx \frac{qW(n'_p - n''_{p0})}{2} = \frac{qWn_{p0}\left(e^{\frac{|q|V_{BE}}{kT}} - e^{\frac{|q|V_{BC}}{kT}}\right)}{2} = 0.8\,10^{-19}\,10^{-4}\left(e^{\frac{0.6}{0.025}} - e^{\frac{-1}{0.025}}\right)\frac{(1.45\,10^{10})^2}{10^{16}} = 4.46\,10^{-9}\,Ccm^{-2}$$

[8]

1E Fields 2004 : Solutions [E1.3]

4)



a) $\int_{surf} D \cdot ds = Q$ bookwork

Since D is uniform, gives:

$DA = Q \quad D = \frac{Q}{A} \quad \therefore \ E = \frac{Q}{\varepsilon_0 A}$

b) $V = -\int E dx = Et = \frac{Qt}{\varepsilon_0 A}$ bookwork

$C = Q/V = \varepsilon_0 A/t$

c) $D = Q/A$ as before

$E = Q/\varepsilon A = \frac{Q}{\varepsilon_0 A(\varepsilon_{r_1} + \Delta\varepsilon_r x/t)}$ $\quad \Delta\varepsilon_r = \varepsilon_{r_2} - \varepsilon_{r_1}$

d) $V = -\int_0^t E dx = \frac{-Q}{\varepsilon_0 A}\int_0^t \frac{dx}{\varepsilon_{r_1} + \frac{\Delta\varepsilon_r}{t} x} = \frac{-Q}{\varepsilon_0 A} \cdot \frac{t}{\Delta\varepsilon_r} \ln\left(\varepsilon_{r_1} + \frac{\Delta\varepsilon_r x}{t}\right)\Big|_0^t$

$|V| = \frac{Qt}{\varepsilon_0 \Delta\varepsilon_r A} \ln\left(\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}\right) \quad C = \frac{Q}{|V|} = \frac{\varepsilon_0 \Delta\varepsilon_r A}{t \ln(\varepsilon_{r_2}/\varepsilon_{r_1})}$

e) $\varepsilon_{r_2} = \varepsilon_{r_1} + \Delta\varepsilon_r$

$C = \frac{\varepsilon_0 \Delta\varepsilon_r A}{t \ln(1 + \Delta\varepsilon_r/\varepsilon_{r_1})}$ $\quad$ if $\Delta\varepsilon_r \ll \varepsilon_{r_1}$, $\ln\left(1 + \frac{\Delta\varepsilon_r}{\varepsilon_{r_1}}\right) \simeq \frac{\Delta\varepsilon_r}{\varepsilon_{r_1}}$

$\therefore \ C \simeq \frac{\varepsilon_0 \Delta\varepsilon_r A}{t(\Delta\varepsilon_r/\varepsilon_{r_1})} = \frac{\varepsilon_0 \varepsilon_{r_1} A}{t}$ ✓

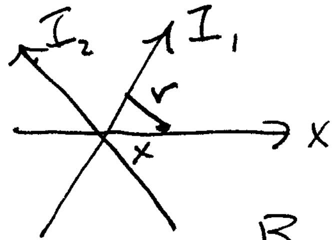
c) - e) : new theoretical application

new theoretical application (whole question)

5) a) By right-hand rule, the +x current gives upwards B for y>0, downwards for y<0, and the +y current gives upwards B for x<0 and down for x>0. For +x, −y, both contributions are down, so B is down into the page.

In the (+,+) and (−,−) quadrants, the contributions from the 2 wires have opposite sense. Along the diagonal x=y, by symmetry, the magnitudes of the contributions are equal so $\vec{B}=0$ along x=y.

b) Everywhere along the x axis the contributions of the 2 wires are equal. Each wire contributes $B = \frac{\mu_0 I}{2\pi r}$.

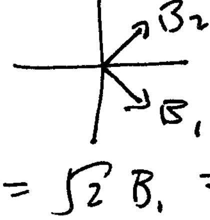


$r^2 + r^2 = x^2 \qquad r = x/\sqrt{2}$

$$B_1 = \frac{\mu_0 I}{2\pi x/\sqrt{2}} = \frac{\sqrt{2}}{2}\frac{\mu_0 I}{\pi x}$$

Total B then is $2B_1 = \frac{\sqrt{2}\mu_0 I}{\pi x}$

Along the z axis both contributions are // to x−y plane and ⊥ the corresponding wire:



With a resultant in the +x direction. (for z>0)

$$B = \sqrt{2}\,B_1 = \sqrt{2}\left(\frac{\mu_0 I}{2\pi z}\right)$$